Today you will:

- Solve systems of linear equations in two variables
  - using substitution
  - using elimination

Examples of Systems of Equations in Two Variables:

$$y = -2x - 9 \quad \text{Equation 1}$$
$$6x - 5y = -19 \quad \text{Equation 2}$$

$$-x + y = 3 \quad \text{Equation 1}$$
$$3x + y = -1 \quad \text{Equation 2}$$

What does it mean to solve Systems of Equations in Two Variables:

Each of the two equations represents a line…

Solving the system means finding if and where these two lines intersect

…in other words you will get an x and a y … (x, y)

Solving by Substitution

Process for solving a System of Linear Equations in two variables by Substitution:

1. Solve one of the equations for one of the variables
2. Substitute the expression from step 1 into the other equation and solve for the other variable
3. Substitute the value from step 2 into one of the original equations and solve
4. Double check your answers by plugin back into both equations!

Example 1: Solve the system of linear equations by substitution.

$y = -2x - 9$ Equation 1

$6x - 5y = -19$ Equation 2

SOLUTION

Step 1 Equation 1 is already solved for y.

Step 2 Substitute $-2x - 9$ for y in Equation 2 and solve for x.

$6x - 5y = -19$ Equation 2

$6x - 5(-2x - 9) = -19$ Substitute $-2x - 9$ for y.

$6x + 10x + 45 = -19$ Distributive Property

$16x + 45 = -19$ Combine like terms.

$16x = -64$ Subtract 45 from each side.

$x = -4$ Divide each side by 16.

Step 3 Substitute -4 for x in Equation 1 and solve for y.

$y = -2x - 9$ Equation 1

$= -2(-4) - 9$ Substitute -4 for x.

$= 8 - 9$ Multiply.

$= -1$ Subtract.

The solution is $(-4, -1)$.

Check

Equation 1

$y = -2x - 9$

$-1 \stackrel{?}{=} -2(-4) - 9$

$-1 = -1$ ✓

Equation 2

$6x - 5y = -19$

$6(-4) - 5(-1) \stackrel{?}{=} -19$

$-19 = -19$ ✓

Example 2: Solve the system of linear equations by substitution.

$$-x + y = 3 \qquad \text{Equation 1}$$
$$3x + y = -1 \qquad \text{Equation 2}$$

SOLUTION

Step 1 Solve for y in Equation 1.

$$y = x + 3 \qquad \text{Revised Equation 1}$$

Step 2 Substitute $x + 3$ for y in Equation 2 and solve for x.

$$3x + y = -1 \qquad \text{Equation 2}$$
$$3x + (x + 3) = -1 \qquad \text{Substitute } x + 3 \text{ for } y.$$
$$4x + 3 = -1 \qquad \text{Combine like terms.}$$
$$4x = -4 \qquad \text{Subtract 3 from each side.}$$
$$x = -1 \qquad \text{Divide each side by 4.}$$

Step 3 Substitute -1 for x in Equation 1 and solve for y.

$$-x + y = 3 \qquad \text{Equation 1}$$
$$-(-1) + y = 3 \qquad \text{Substitute } -1 \text{ for } x.$$
$$y = 2 \qquad \text{Subtract 1 from each side.}$$

The solution is $(-1, 2)$.

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Algebraic Check

Equation 1

$$-x + y = 3$$

$$-(-1) + 2 \stackrel{?}{=} 3$$

$$3 = 3 \checkmark$$

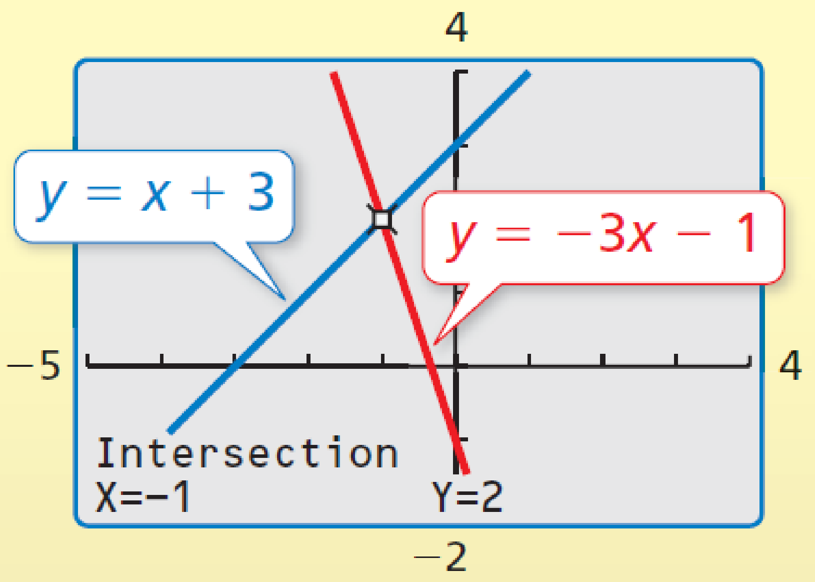
Equation 2

$$3x + y = -1$$

$$3(-1) + 2 \stackrel{?}{=} -1$$

$$-1 = -1 \checkmark$$

Graphical Check



A drama club earns \$1040 from a production. A total of 64 adult tickets and 132 student tickets are sold. An adult ticket costs twice as much as a student ticket. Write a system of linear equations that represents this situation. What is the price of each type of ticket?

SOLUTION

1. Understand the Problem You know the amount earned, the total numbers of adult and student tickets sold, and the relationship between the price of an adult ticket and the price of a student ticket. You are asked to write a system of linear equations that represents the situation and find the price of each type of ticket.

Words 64 • Adult ticket price + 132 • Student ticket price = 1040

Adult ticket price = 2 • Student ticket price

Variables Let *x* be the price (in dollars) of an adult ticket and let *y* be the price (in dollars) of a student ticket.

System

System $64x + 132y = 1040$ Equation 1

$x = 2y$ Equation 2

Step 1 Equation 2 is already solved for x.

Step 2 Substitute $2y$ for x in Equation 1 and solve for y.

$$64x + 132y = 1040 \quad \text{Equation 1}$$

$$64(2y) + 132y = 1040 \quad \text{Substitute } 2y \text{ for } x.$$

$$260y = 1040 \quad \text{Simplify.}$$

$$y = 4 \quad \text{Simplify.}$$

STUDY TIP

You can use either of the original equations to solve for x. However, using Equation 2 requires fewer calculations.

Step 3 Substitute 4 for y in Equation 2 and solve for x.

$$x = 2y \quad \text{Equation 2}$$

$$x = 2(4) \quad \text{Substitute 4 for } y.$$

$$x = 8 \quad \text{Simplify.}$$

The solution is (8, 4). So, an adult ticket costs \$8 and a student ticket costs \$4.

4. Look Back To check that your solution is correct, substitute the values of x and y into both of the original equations and simplify.

$$64(8) + 132(4) = 1040 \qquad 8 = 2(4)$$

$$1040 = 1040 \checkmark \qquad 8 = 8 \checkmark$$

Solving by Elimination

Process for solving a System of Linear Equations in two variables by Elimination:

1. Multiply, if necessary, one or both of equations by a constant so at least one pair of like terms has the same or opposite coefficients

2. Add or subtract the equations to eliminate one of the variables

3. Solve the resulting equation

4. Substitute the value from step 3 into one of the original equations and solve for the other variable

5. Double check your answers by plugin back into both equations!

Example 1: Solve the system of linear equations by elimination.

$$3x + 2y = 4 \quad \text{Equation 1}$$
$$3x - 2y = -4 \quad \text{Equation 2}$$

SOLUTION

Step 1 Because the coefficients of the *y*-terms are opposites, you do not need to multiply either equation by a constant.

Step 2 Add the equations.

$$\begin{array}{rll} 3x + 2y &= 4 & \text{Equation 1} \\ 3x - 2y &= -4 & \text{Equation 2} \\ \hline 6x \quad\quad &= & \text{Add the equations.} \end{array}$$

Step 3 Solve for *x*.

$$6x = 0 \quad \text{Resulting equation from Step 2}$$
$$x = 0 \quad \text{Divide each side by 6.}$$

Step 4 Substitute 0 for *x* in one of the original equations and solve for *y*.

$$3x + 2y = 4 \quad \text{Equation 1}$$
$$3(0) + 2y = 4 \quad \text{Substitute 0 for } x.$$
$$y = 2 \quad \text{Solve for } y.$$

The solution is (0, 2).

Check

Equation 1

$$3x + 2y = 4$$
$$3(0) + 2(2) \stackrel{?}{=} 4$$
$$4 = 4 \checkmark$$

Equation 2

$$3x - 2y = -4$$
$$3(0) - 2(2) \stackrel{?}{=} -4$$
$$-4 = -4 \checkmark$$

Example 2: Solve the system of linear equations by elimination.

$$-10x + 3y = 1 \quad \text{Equation 1}$$
$$-5x - 6y = 23 \quad \text{Equation 2}$$

SOLUTION

Step 1 Multiply Equation 2 by -2 so that the coefficients of the x-terms are opposites.

$-10x + 3y = 1$ → $-10x + 3y = 1$ Equation 1

$-5x - 6y = 23$ **Multiply by -2.** → $10x + 12y = -46$ Revised Equation 2

Step 2 Add the equations.

$$\begin{array}{rl} -10x + 3y = 1 & \text{Equation 1} \\ 10x + 12y = -46 & \text{Revised Equation 2} \\ \hline 15y & \text{Add the equations.} \end{array}$$

Step 3 Solve for y.

$$15y = -45 \quad \text{Resulting equation from Step 2}$$
$$y = -3 \quad \text{Divide each side by 15.}$$

Step 4 Substitute -3 for y in one of the original equations and solve for x.

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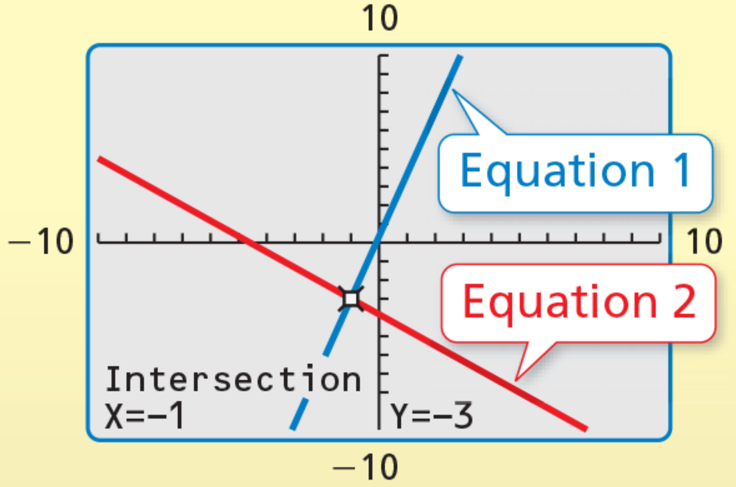
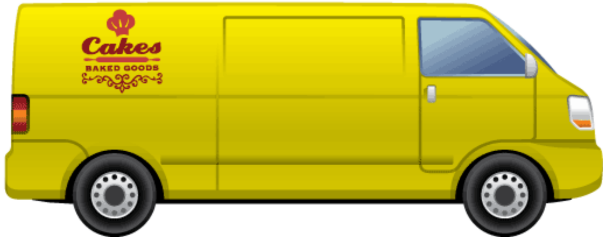
$$-5x - 6y = 23 \qquad \text{Equation 2}$$
$$-5x - 6(-3) = 23 \qquad \text{Substitute } -3 \text{ for } y.$$
$$-5x + 18 = 23 \qquad \text{Multiply.}$$
$$-5x = 5 \qquad \text{Subtract 18 from each side.}$$
$$x = -1 \qquad \text{Divide each side by } -5.$$

Check



The solution is $(-1, -3)$.



A business with two locations buys seven large delivery vans and five small delivery vans. Location A receives five large vans and two small vans for a total cost of $235,000. Location B receives two large vans and three small vans for a total cost of $160,000. What is the cost of each type of van?

SOLUTION

1. **Understand the Problem** You know how many of each type of van each location receives. You also know the total cost of the vans for each location. You are asked to find the cost of each type of van.

Words	5 •	Cost of large van	+ 2 •	Cost of small van	= 235,000
	2 •	Cost of large van	+ 3 •	Cost of small van	= 160,000

Variables

System $5x + 2y = 235{,}000$ Equation 1

$2x + 3y = 160{,}000$ Equation 2

System $5x + 2y = 235{,}000$ Equation 1

$2x + 3y = 160{,}000$ Equation 2

Step 1 Multiply Equation 1 by -3. Multiply Equation 2 by 2.

$5x + 2y = 235{,}000$ **Multiply by -3.** $-15x - 6y = -705{,}000$ Revised Equation 1

$2x + 3y = 160{,}000$ **Multiply by 2.** $4x + 6y = 320{,}000$ Revised Equation 2

Step 2 Add the equations.

$$
\begin{array}{rcrl}
-15x - 6y & = & -705{,}000 & \text{Revised Equation 1} \\
4x + 6y & = & 320{,}000 & \text{Revised Equation 2} \\
\hline
-11x & & & \text{Add the equations.}
\end{array}
$$

Step 3 Solving the equation $-11x = -385{,}000$ gives $x = 35{,}000$.

Step 4 Substitute 35,000 for x in one of the original equations and solve for y.

$$
\begin{array}{rcll}
5x + 2y & = & 235{,}000 & \text{Equation 1} \\
5(35{,}000) + 2y & = & 235{,}000 & \text{Substitute 35,000 for } x. \\
y & = & 30{,}000 & \text{Solve for } y.
\end{array}
$$

The solution is (35,000, 30,000). So, a large van costs \$35,000 and a small van costs \$30,000.

The solution is (35,000, 30,000). So, a large van costs \$35,000 and a small van costs \$30,000.

4. **Look Back** Check to make sure your solution makes sense with the given information. For Location A, the total cost is $5(35{,}000) + 2(30{,}000) = \$235{,}000$. For Location B, the total cost is $2(35{,}000) + 3(30{,}000) = \$160{,}000$. So, the solution makes sense.